edexcel

Mark Scheme (Results)

January 2016

Pearson Edexcel International A Level in Further Pure Mathematics 1 (WFM01/01)

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January 2016
Publications Code IA043231

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - $\ast$ The answer is printed on the paper or ag- answer given
   - $\llcorner$ or d… The second mark is dependent on gaining the first mark

4. All A marks are ‘correct answer only’ (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = ...

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = ...

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

January 2016
WFM01 Further Pure Mathematics F1
Mark Scheme

Question Number	Scheme	Notes	Marks
1. (a)	$\{(3+2\mathrm{i})(1-\mathrm{i})\}=3-3\mathrm{i}+2\mathrm{i}+2$	At least 3 correct terms	M1
	$=5-\mathrm{i}$	**cao** (Correct answer **only** scores both marks)	A1
			(2)
(b)	$w^*=1+\mathrm{i}$	Understanding that $w^*=1+\mathrm{i}$	B1
	$\left\{\frac{z}{w^*}=\right\}\frac{3+2\mathrm{i}}{1+\mathrm{i}}\times\frac{1-\mathrm{i}}{1-\mathrm{i}}$	Multiplies top and bottom by the conjugate of the denominator	M1
	$\left\{=\frac{3-3\mathrm{i}+2\mathrm{i}+2}{1+1}\right\}=\frac{5}{2}-\frac{1}{2}\mathrm{i}$	$\frac{5}{2}-\frac{1}{2}\mathrm{i}$ or $2.5-0.5\mathrm{i}$	A1
			(3)
(c)	$\left\{\lvert 3+2\mathrm{i}+k\rvert=\sqrt{53}\Rightarrow\right\}\ (3+k)^2+4;=53$	Substitutes for z and uses Pythagoras correctly.	M1;
		Correct equation in any form	A1
	$(3+k)^2+4=53\Rightarrow(3+k)^2=49\Rightarrow k=$ or $(3+k)^2+4=53\Rightarrow k^2+6k-40=0$ $\Rightarrow(k-4)(k+10)=0\Rightarrow k=$	**dependent on the previous M mark** Attempt to solve for k	dM1
	$\{k=\}\ 4,\ -10$	Both $\{k=\}\ 4\ ,\ -10$	A1
			(4)
			9

Question 1 Notes

1. (b)	**Note**	<u>**Alternative acceptable method:**</u> $\left(\frac{z}{w^*}\right)\left(\frac{w}{w}\right)=\frac{zw}{\lvert w\rvert^2}=\frac{5-\mathrm{i}}{2}=\frac{5}{2}-\frac{1}{2}\mathrm{i}$
(b)	**Note**	Give A0 for writing down $\frac{5-\mathrm{i}}{2}$ without reference to $\frac{5}{2}-\frac{1}{2}\mathrm{i}$ or $2.5-0.5\mathrm{i}$
	Note	Give B0M0A0 for writing down $\frac{5}{2}-\frac{1}{2}\mathrm{i}$ from no working in part (b).
	Note	Give B0M1A0 for $\frac{3+2\mathrm{i}}{1-\mathrm{i}}\times\frac{1+\mathrm{i}}{1+\mathrm{i}}$
	Note	Simplifying a correct $\frac{5}{2}-\frac{1}{2}\mathrm{i}$ in part (b) to a final answer of $5-\mathrm{i}$ is A0
(c)	**Note**	Give final A0 if a candidate rejects one of $k=4$ or $k=-10$
(b)	**ALT**	$\frac{3+2\mathrm{i}}{1+\mathrm{i}}=a+b\mathrm{i}$ **B1**; $\Rightarrow 3+2\mathrm{i}=(a+b\mathrm{i})(1+\mathrm{i})\Rightarrow 3=a-b,\ 2=a+b\Rightarrow a=...,\ b=...$ for **M1** and $\frac{5}{2}-\frac{1}{2}\mathrm{i}$ for **A1**

Question Number	Scheme	Notes	Marks
2.	$\mathrm{f}(x)=x^2-\dfrac{3}{\sqrt{x}}-\dfrac{4}{3x^2}$		
(a)	$\mathrm{f}(1.6)=-0.3325....$ $\mathrm{f}(1.7)=0.1277....$	Attempts to evaluate both $\mathrm{f}(1.6)$ and $\mathrm{f}(1.7)$ and either f(1.6) = awrt –0.3 or f(1.7) = awrt 0.1	M1
	Sign change (positive, negative) (and $\mathrm{f}(x)$ is continuous) therefore (a root) α is between $x=1.6$ and $x=1.7$	Both f(1.6) = awrt –0.3 and f(1.7) = awrt 0.1, sign change and conclusion.	A1 **cso**
			(2)
(b)	$\mathrm{f}'(x)=2x+\dfrac{3}{2}x^{-\frac{3}{2}}+\dfrac{8}{3}x^{-3}$	At least one of either $x^2\to\pm Ax$ or $-\dfrac{3}{\sqrt{x}}\to\pm Bx^{-\frac{3}{2}}$ or $-\dfrac{4}{3x^2}\to\pm Cx^{-3}$ where *A*, *B* and *C* are non-zero constants.	M1
		At least 2 differentiated terms are correct	A1
		Correct differentiation	A1
	$\left\{\alpha\simeq1.6-\dfrac{\mathrm{f}(1.6)}{\mathrm{f}'(1.6)}\right\}\Rightarrow\alpha\simeq1.6-\dfrac{-0.332541....}{4.592200....}$	**dependent on the previous M mark** Valid attempt at Newton-Raphson using their values of $\mathrm{f}(1.6)$ and $\mathrm{f}'(1.6)$	dM1
	$\{\alpha=1.672414...\Rightarrow\}\ \alpha=1.672$	**dependent on all 4 previous marks** 1.672 ***on their first iteration*** (Ignore any subsequent applications)	A1 **cso cao**
	Correct derivative followed by correct answer scores full marks in (b) **Correct answer with <u>no</u> working scores no marks in (b)**		
			(5)
			7

		Question 2 Notes
2. (a)	A1	**correct solution only.** Candidate needs to state both f(1.6) = awrt –0.3 **and** f(1.7) = awrt 0.1 along with **a reason and conclusion.** Reference to change of sign **or** $\mathrm{f}(1.6)\times\mathrm{f}(1.7)<0$ **or** a diagram **or** < 0 and > 0 **or** one positive, one negative are sufficient reasons. There must be a (minimal, not incorrect) conclusion, eg. root is in between 1.6 and 1.7, hence root is in interval, QED and a square are all acceptable. Ignore the presence or absence of any reference to continuity. A minimal acceptable reason and conclusion is "change of sign, hence root".
(b)	**Note**	Incorrect differentiation followed by their estimate of α with no evidence of applying the NR formula is final dM0A0.
	Note	If the answer is incorrect it must be clear that we must see evidence of both $\mathrm{f}(1.6)$ and $\mathrm{f}'(1.6)$ being used in the Newton-Raphson process. So that just $1.6-\dfrac{\mathrm{f}(1.6)}{\mathrm{f}'(1.6)}$ with an incorrect answer and no other evidence scores M0.

Question Number	Scheme	Notes	Marks
3.	$x^2 - 2x + 3 = 0$		
(a) (i)	$\alpha + \beta = 2,\quad \alpha\beta = 3$	Both $\alpha + \beta = 2,\quad \alpha\beta = 3$	B1
(ii)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \ldots\ldots$	**Use** of a **correct** identity for $\alpha^2 + \beta^2$ (May be implied by their work)	M1
	$= 2^2 - 6 = -2$ *	-2 **from a correct solution only**	A1 *
(iii)	$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = \ldots\ldots$ or $= (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta) = \ldots$	**Use** of a **correct** identity for $\alpha^3 + \beta^3$ (May be implied by their work)	M1
	$= 8 - 3(3)(2) = -10$ or $= 2(-2-3) = -10$	-10 **from a correct solution only**	A1
			(5)
(b)(i)	$(\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2 = \alpha^4 + 2(\alpha\beta)^2 + \beta^4 - 2(\alpha\beta)^2 = \alpha^4 + \beta^4$	Correct algebraic proof	B1 *
(ii)	Sum $= \alpha^3 + \beta^3 - (\alpha + \beta) = -10 - 2 = -12$	Correct working without using explicit roots leading to a correct sum.	B1
	Product $= (\alpha^3 - \beta)(\beta^3 - \alpha) = (\alpha\beta)^3 - (\alpha^4 + \beta^4) + \alpha\beta$	Attempts to expand giving at least one term	M1
	$= (\alpha\beta)^3 - ((\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2) + \alpha\beta$		
	$= 27 - (4 - 18) + 3 = 44$	Correct product	A1
	$\{x^2 - \text{sum}\,x + \text{product} = 0 \Rightarrow\}\ x^2 + 12x + 44 = 0$	Applying $x^2 - (\text{sum})x + \text{product}$	M1
		$x^2 + 12x + 44 = 0$	A1
			(6)
			11

		Question 3 Notes
(a) (i)	**1st A1**	$\alpha + \beta = -2,\quad \alpha\beta = 3 \Rightarrow \alpha^2 + \beta^2 = 4 - 6 = -2$ is M1A0 cso
(b) (ii)	**1st A1**	$\alpha + \beta = -2,\quad \alpha\beta = 3 \Rightarrow (\alpha\beta)^3 - (\alpha^4 + \beta^4) + \alpha\beta = 44$ is first M1A1
(a)	**Note**	Applying $1+\sqrt{2}\,\mathrm{i}, 1-\sqrt{2}\,\mathrm{i}$ explicitly in part (a) will score B0M0A0M0A0
(b)	**Note**	Applying $1+\sqrt{2}\,\mathrm{i}, 1-\sqrt{2}\,\mathrm{i}$ explicitly in part (b) will score a maximum of B1B0M0A0M1A0
(a)	**Note**	Finding $\alpha + \beta = 2, \alpha\beta = 3$ by writing down or applying $1+\sqrt{2}\,\mathrm{i}, 1-\sqrt{2}\,\mathrm{i}$ but then writing $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 2^2 - 6 = -2$ and $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = 8 - 3(3)(2) = -10$ scores B0M1A0M1A0 in part (a). Such candidates will be able to score all marks in part (b) if they use the method as detailed on the scheme in part (b).
(b)(ii)	**Note**	A correct method leading to a candidate stating $p = 1, q = 12, r = 44$ without writing a final answer of $x^2 + 12x + 44 = 0$ is **final** M1A0

Question Number	Scheme	Notes	Marks
4. (a)	Rotation	Rotation	B1
	225 degrees (anticlockwise)	225 degrees or $\frac{5\pi}{4}$ (anticlockwise) or 135 degrees clockwise	B1 o.e.
	about (0, 0)	**This mark is dependent on at least one of the previous B marks being awarded.** About (0, 0) or about *O* or about the origin	dB1
	Note: Give 2nd B0 for 225 degrees clockwise		**(3)**
(b)	$\{n=\}\ 8$	8	B1 **cao**
			(1)
(c) **Way 1**	$\mathbf{A}^{-1} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$ or $\begin{pmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{pmatrix}$	Correct matrix	B1
	$\{\mathbf{B} = \mathbf{CA}^{-1}\} = \begin{pmatrix} 2 & 4 \\ -3 & -5 \end{pmatrix}\begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} = \ldots$	Attempts $\mathbf{CA}^{-1}$ and finds at least one element of the matrix **B**	M1
	$= \begin{pmatrix} \sqrt{2} & -3\sqrt{2} \\ -\sqrt{2} & 4\sqrt{2} \end{pmatrix}$	**dependent on the previous B1M1 marks** At least 2 correct elements	A1
		All elements are correct	A1
			(4)
(c) **Way 2**	$\{\mathbf{BA} =\}\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ -3 & -5 \end{pmatrix}$	Correct statement using 2×2 matrices. All 3 matrices must contain four elements. **(Can be implied).** (Allow one slip in copying down **C**)	B1
	$-\frac{a}{\sqrt{2}} - \frac{b}{\sqrt{2}} = 2,\ \frac{a}{\sqrt{2}} - \frac{b}{\sqrt{2}} = 4$ or $-\frac{c}{\sqrt{2}} - \frac{d}{\sqrt{2}} = -3,\ \frac{c}{\sqrt{2}} - \frac{d}{\sqrt{2}} = -5$ and finds at least one of either *a* or *b* or *c* or *d*	Applies $\mathbf{BA} = \mathbf{C}$ and attempts simultaneous equations in *a* and *b* **or** *c* and *d* **and** finds at least one of either *a* or *b* or *c* or *d*	M1
	$= \begin{pmatrix} \sqrt{2} & -3\sqrt{2} \\ -\sqrt{2} & 4\sqrt{2} \end{pmatrix}$ or $a=\sqrt{2}, b=-3\sqrt{2}, c=-\sqrt{2}, d=4\sqrt{2}$	**dependent on the previous B1M1 marks** At least 2 correct elements	A1
		All elements are correct	A1
			(4)
			8

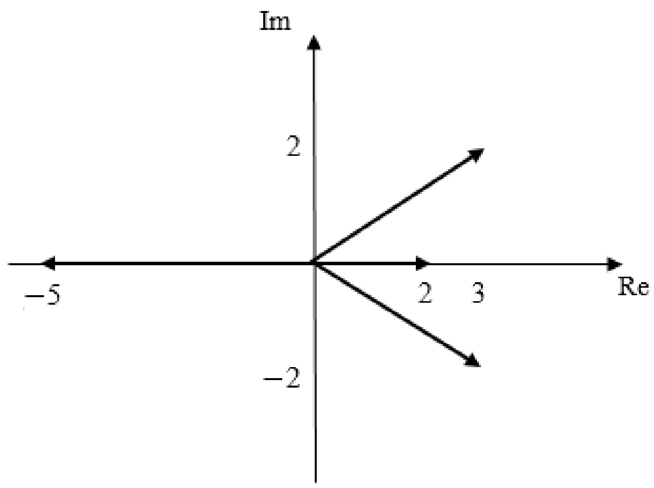
Question 4 Notes

4. (a)	**Note**	Condone "Turn" for the 1st B1 mark.
(c)	**Note**	You can ignore previous working prior to a candidate finding $\mathbf{CA}^{-1}$ (i.e. you can ignore the statements $\mathbf{C} = \mathbf{BA}$ or $\mathbf{C} = \mathbf{AB}$).
	A1 A1	You can allow equivalent matrices/values, e.g. $\begin{pmatrix} \frac{2}{\sqrt{2}} & -\frac{6}{\sqrt{2}} \\ -\frac{2}{\sqrt{2}} & \frac{8}{\sqrt{2}} \end{pmatrix}$

Question Number	Scheme	Notes	Marks
5. (a)	$\left\{\sum_{r=1}^{n} 8r^3 - 3r\right\} = 8\left(\frac{1}{4}n^2(n+1)^2\right) - 3\left(\frac{1}{2}n(n+1)\right)$	Attempt to substitute at least one of the standard formulae correctly into the given expression	M1
		Correct expression	A1
	$= \frac{1}{2}n(n+1)\left[4n(n+1) - 3\right]$	**dependent on the previous M mark** Attempt to factorise at least $n(n + 1)$ having used both standard formulae correctly	dM1
	$= \frac{1}{2}n(n+1)\left[4n^2 + 4n - 3\right]$	{*this step does not have to be written*}	
	$= \frac{1}{2}n(n+1)(2n+3)(2n-1)$	Correct completion with no errors	A1 **cso**
			(4)
(b)	Let $f(n) = \frac{1}{2}n(n+1)(2n+3)(2n-1)$, $g(n) = \frac{8}{4}n^2(n+1)^2$ & $h(n) = \pm\frac{3}{2}n(n+1)$		
	$\left\{\sum_{r=5}^{10} 8r^3 - 3r\right\} = \frac{1}{2}(10)(11)(23)(19) - \frac{1}{2}(4)(5)(11)(7)$ $\{= 24035 - 770 = 23265\}$	Attempts to find either • $f(10)$ **and** $f(4)$ or $f(5)$ • $g(10)$ **and** $g(4)$ or $g(5)$ **and** $h(10)$ **and** $h(4)$ or $h(5)$	M1
	$\sum_{r=5}^{10} kr^2 = k\left(\frac{1}{6}(10)(11)(21) - \frac{1}{6}(4)(5)(9)\right)\{= k(385 - 30) = 355k\}$ or $= k\left(5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2\right)\{= 355k\}$	Correct attempt at $\sum_{r=5}^{10} kr^2$	M1
	$23265 + 355k = 22768 \Rightarrow k = -\frac{497}{355}$ or $-\frac{7}{5}$	**dependent on both previous M marks.** Uses both previous method mark results to form a linear equation in k using 22768 and solves to give $k = ...$	ddM1
		$k = -\frac{497}{355}$ or $-\frac{7}{5}$ or -1.4 or equivalent	A1 o.e.
			(4)
			8

		Question 5 Notes
5. (a)	**Note**	Applying eg. $n = 1$, $n = 2$ to the printed equation without applying the standard formula to give $a = 2$, $b = -1$ is M0A0M0A0
	Alt **dM1** **A1 cso**	**Alternative Method:** Using $2n^4 + 4n^3 + \frac{1}{2}n^2 - \frac{3}{2}n \equiv an^4 + (b + \frac{5}{2}a)n^3 + (\frac{5}{2}b + \frac{3}{2}a)n^2 + \frac{3}{2}bn$ o.e. Equating coefficients to give both $a = 2$, $b = -1$ Demonstrates that the identity works for **all** of its terms
(b)	**Note**	$f(10) - f(5) = \frac{1}{2}(10)(11)(23)(19) - \frac{1}{2}(5)(6)(13)(9)\ \{= 24035 - 1755 = 22280\}$
	Note	Applying $\sum_{r=5}^{10} 8r^3 - \sum_{r=5}^{10} 3r + k\sum_{r=5}^{10} r^2$ gives either • $(24200 - 165 + 385k) - (800 - 30 + 30k) = 22768$ • $23400 - 135 + 355k = 22768$
	Note	$985 + 25k + 1710 + 36k + 2723 + 49k + 4072 + 64k + 5805 + 81k + 7970 + 100k = 23265 + 355k$ is fine for the first two M1M1 marks with the final ddM1A1 leading to $k = -1.4$

Question Number	Scheme	Notes	Marks
6. (a)	$y = \frac{c^2}{x} = c^2x^{-1} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -c^2x^{-2} = -\frac{c^2}{x^2}$	$\frac{\mathrm{d}y}{\mathrm{d}x} = k\,x^{-2}$	
	$xy = c^2 \Rightarrow x\frac{\mathrm{d}y}{\mathrm{d}x} + y = 0$	Correct use of product rule. The sum of two terms, one of which is correct.	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}p}.\frac{\mathrm{d}p}{\mathrm{d}x} = -\frac{c}{p^2}.\frac{1}{c}$	their $\frac{\mathrm{d}y}{\mathrm{d}p} \times \frac{1}{\text{their } \frac{\mathrm{d}x}{\mathrm{d}p}}$	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -c^2x^{-2}$ or $x\frac{\mathrm{d}y}{\mathrm{d}x} + y = 0$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{c}{p^2}.\frac{1}{c}$	Correct differentiation	A1
	So, $m_N = p^2$	Perpendicular gradient rule where $m_N (\neq m_T)$ is found from using calculus.	M1
	$y - \frac{c}{p} = p^2(x - cp)$ or $y = p^2x + \frac{c}{p} - cp^3$	Correct line method where m_N is found from using calculus.	M1
	$py - p^3x = c(1 - p^4)$ *		A1*
			(5)
(b)	$y = \frac{c^2}{x} \Rightarrow p\frac{c^2}{x} - p^3x = c(1-p^4)$ or $x = \frac{c^2}{y} \Rightarrow py - p^3\frac{c^2}{y} = c(1-p^4)$ Substitutes $y = \frac{c^2}{p}$ or $x = \frac{c^2}{y}$ into the printed equation to obtain an equation in either x, c and p only or in y, c and p only.		M1
	$p^3x^2 + c(1-p^4)x - c^2p = 0$ or $py^2 - c(1-p^4)y - c^2p^3 = 0$		
	$(x - cp)(p^3x + c) = 0 \Rightarrow x = \ldots$ or $\left(y - \frac{c}{p}\right)(yp + cp^4) = 0 \Rightarrow y = \ldots$ Correct attempt of solving a 3TQ to find the x or y coordinate of Q		M1
	$Q\left(-\frac{c}{p^3}, -cp^3\right)$ — Can be simplified or un-simplified.	At least one correct coordinate.	A1
		Both correct coordinates	A1
	Note: If Q is stated as coordinates then they must be correct for the final A1 mark.		**(4)**
(b) **ALT**	Let Q be $\left(cq, \frac{c}{q}\right)$ so $\frac{c}{q}p - p^3cq = c(1-p^4)$ Substitutes $x = cq$ or $y = \frac{c}{q}$ into the printed equation to obtain an equation in only p, c and q.		M1
	$cp - p^3cq^2 = cq - cqp^4 \Rightarrow p - q - p^3q^2 + qp^4 = 0$		
	$(p - q)(1 + p^3q) = 0 \Rightarrow q = \ldots$ Correct attempt to find q in terms of p		M1
	$Q\left(-\frac{c}{p^3}, -cp^3\right)$ — Can be simplified or un-simplified.	At least one correct coordinate	A1
		Both correct coordinates	A1
			(4)
			9

Question Number	Scheme	Notes	Marks
7.	$\mathrm{f}(x) = x^4 - 3x^3 - 15x^2 + 99x - 130$		
(a)	3 – 2i is also a root	3 – 2i	B1
	$x^2 - 6x + 13$	Attempt to expand $(x-(3+2\mathrm{i}))(x-(3-2\mathrm{i}))$ or any valid method to establish the quadratic factor e.g. $x = 3 \pm 2\mathrm{i} \Rightarrow x - 3 = \pm 2\mathrm{i} \Rightarrow x^2 - 6x + 9 = -4$ or sum of roots 6, product of roots 13	M1
		$x^2 - 6x + 13$	A1
	$\mathrm{f}(x) = \left(x^2 - 6x + 13\right)\left(x^2 + 3x - 10\right)$	Attempt other quadratic factor. **Note:** Using long division to get as far as $x^2 \pm kx$ is fine for this mark.	M1
		$x^2 + 3x - 10$	A1
	$\left\{x^2 + 3x - 10 =\right\} (x+5)(x-2) \Rightarrow x = \ldots$	Correct method for solving a 3TQ on their 2nd quadratic factor	M1
	$x = -5,\ x = 2$	Both values correct	A1
			(7)
	Note: Writing down $2,\ -5,\ 3+2\mathrm{i},\ 3-2\mathrm{i}$ with **no** working is B1M0A0M0A0M0A0		
(a)	**<u>Alternative using Factor Theorem</u>**		
	3 – 2i	3 – 2i	B1
	$\left\{\mathrm{f}(2) =\right\} 2^4 - 3 \times 2^3 - 15 \times 2^2 + 99 \times 2 - 130 = 0$	Attempts to find $\mathrm{f}(2)$	M1
		Shows that $\mathrm{f}(2) = 0$	A1
	$\left\{\mathrm{f}(-5) =\right\} (-5)^4 - 3(-5)^3 - 15(-5)^2 + 99 \times (-5) - 130 = 0$	Attempts to find $\mathrm{f}(-5)$	M1
		Shows that $\mathrm{f}(-5) = 0$	A1
	$x = 2,\ x = -5$	**Either** shows that $\mathrm{f}(2) = 0$ and states $x = 2$ **or** shows that $\mathrm{f}(-5) = 0$ and states $x = -5$	M1
		Shows both $\mathrm{f}(2) = 0$ & $\mathrm{f}(-5) = 0$ **and** states both $x = -5,\ x = 2$	A1
			(7)
(b)		• $3 \pm 2\mathrm{i}$ plotted correctly in quadrants 1 and 4 with some evidence of symmetry • **dependent on the final M mark being awarded in part (a).** Their other two roots plotted correctly.	
		Satisfies at least one of the criteria.	B1ft
		Satisfies both criteria with some indication of scale or coordinates stated. All points (arrows) must be in the correct positions relative to each other.	B1ft
			(2)
			9

Question Number	Scheme	Notes	Marks
8.	$S(a,0),\ B(q,r),\ C\left(-a, -\dfrac{2ar}{q-a}\right)$ or $C(-a, -3ar)$		
(a)	$m = \dfrac{r-0}{q-a}$	Correct gradient using $(a, 0)$ and (q, r) (Can be implied)	B1
	• $y = \dfrac{r}{q-a}(x-a)$ or • $y - r = \dfrac{r}{q-a}(x-q)$ • $0 = \dfrac{ra}{q-a} + "c" \Rightarrow "c" = -\dfrac{ra}{q-a}$ and $y = \dfrac{r}{q-a}x - \dfrac{ra}{q-a}$	Correct straight line method	M1
	leading to $(q-a)y = r(x-a)$*	cso	A1*
			(3)
(b)	$C\left(\{-a\}, -\dfrac{2ar}{q-a}\right)$ or height $OCS = \dfrac{2ar}{q-a}$	$-\dfrac{2ar}{q-a}$ or $\dfrac{2ar}{q-a}$	B1
	$\dfrac{2ar}{q-a} = 3r$ or $\dfrac{1}{2}(a)\left(\dfrac{2ar}{q-a}\right) = 3\left(\dfrac{1}{2}\right)(a)(r) \Rightarrow ...$	Applies height OCS $= 3r$ or applies $\text{Area}(OSC) = 3\text{Area}(OSB)$ and rearranges to give $\lambda a = \mu q$ where λ, μ are numerical values.	M1
	$\Rightarrow 5a = 3q$	$5a = 3q$ or $a = \dfrac{3}{5}q$	A1
	$\text{Area}(OBC) = 4\left(\dfrac{1}{2}\right)\left(\dfrac{3q}{5}\right)r$ or $= \left(\dfrac{1}{2}\right)\left(\dfrac{3q}{5}\right)r + \left(\dfrac{3}{2}\right)\left(\dfrac{3q}{5}\right)r$	**dependent on the previous M mark** Uses their $a = \dfrac{3}{5}q$ and applies a correct method to find $\text{Area}(OBC)$ in terms of only q and r	dM1
	$= \dfrac{6}{5}qr$ (*)	$\dfrac{6}{5}qr$	A1* **cso**
			(5)
			8
	<u>Alternative Method (Similar Triangles)</u>		
(b)	$\dfrac{3r}{2a} = \dfrac{r}{q-a}$	$\dfrac{3r}{2a} = \dfrac{r}{q-a}$ or equivalent	B1
	$\dfrac{3r}{2a} = \dfrac{r}{q-a} \Rightarrow ...$	$\dfrac{3r}{2a} = \dfrac{r}{q-a}$ or equivalent and rearranges to give $\lambda a = \mu q$ where λ, μ are numerical values.	M1
	… then apply the original mark scheme.		

Question 8 Notes

8. (a)	**Note**	The first two marks B1M1 can be gained together by applying the formula $\dfrac{y-y_1}{y_2-y_1} = \dfrac{x-x_1}{x_2-x_1}$
		to give $\dfrac{y-0}{r-0} = \dfrac{x-a}{q-a}$
(b)	**Note**	If a candidate uses either $-\dfrac{2ar}{q-a}$ or $-3r$ they can get 1st M1 but not 2nd M1 in (b).

Question Number	Scheme	Notes	Marks
9.	$f(n) = 4^{n+1} + 5^{2n-1}$		
	$f(1) = 4^2 + 5 = 21$	f(1) = 21 is the minimum	B1
	$f(k+1) - f(k) = 4^{k+2} + 5^{2(k+1)-1} - (4^{k+1} + 5^{2k-1})$	Attempts $f(k+1) - f(k)$	M1
	$f(k+1) - f(k) = 3(4^{k+1}) + 24(5^{2k-1})$		
	$= 3(4^{k+1} + 5^{2k-1}) + 21(5^{2k-1})$ or $= 24(4^{k+1} + 5^{2k-1}) - 21(4^{k+1})$	Either $3(4^{k+1} + 5^{2k-1})$ or $3f(k)$; $21(5^{2k-1})$ $24(4^{k+1} + 5^{2k-1})$ or $24f(k)$; $-21(4^{k+1})$	A1; A1
	$f(k+1) = 3f(k) + 21(5^{2k-1}) + f(k)$ or $f(k+1) = 24f(k) - 21(4^{k+1}) + f(k)$	**dependent on at least one of the previous accuracy marks being awarded.** Makes $f(k+1)$ the subject	dM1
	If the result is true for $n = k$, then it is true for $n = k+1$, As the result has been shown to be true for $n = 1$, then the result is is true for all $n \in \mathbb{Z}^+$.		A1 **cso**
			(6)
			6
WAY 2	**General Method:** Using $f(k+1) - mf(k)$		
	$f(1) = 4^2 + 5 = 21$	f(1) = 21 is the minimum	B1
	$f(k+1) - mf(k) = 4^{k+2} + 5^{2(k+1)-1} - m(4^{k+1} + 5^{2k-1})$	Attempts $f(k+1) - f(k)$	M1
	$f(k+1) - mf(k) = (4-m)(4^{k+1}) + (25-m)(5^{2k-1})$		
	$= (4-m)(4^{k+1} + 5^{2k-1}) + 21(5^{2k-1})$ or $= (25-m)(4^{k+1} + 5^{2k-1}) - 21(4^{k+1})$	$(4-m)(4^{k+1} + 5^{2k-1})$ or $(4-m)f(k)$; $21(5^{2k-1})$ $(25-m)(4^{k+1} + 5^{2k-1})$ or $(25-m)f(k)$; $-21(4^{k+1})$	A1; A1
	$f(k+1) = (4-m)f(k) + 21(5^{2k-1}) + mf(k)$ or $f(k+1) = (25-m)f(k) - 21(4^{k+1}) + mf(k)$	**dependent on at least one of the previous accuracy marks being awarded.** Makes $f(k+1)$ the subject	dM1
	If the result is true for $n = k$, then it is true for $n = k+1$, As the result has been shown to be true for $n = 1$, then the result is is true for all $n \in \mathbb{Z}^+$.		A1 **cso**
WAY 3	$f(1) = 4^2 + 5 = 21$	f(1) = 21 is the minimum	B1
	$f(k+1) = 4^{k+2} + 5^{2(k+1)-1}$	Attempts $f(k+1)$	M1
	$f(k+1) = 4(4^{k+1}) + 25(5^{2k-1})$		
	$= 4(4^{k+1} + 5^{2k-1}) + 21(5^{2k-1})$ or $= 25(4^{k+1} + 5^{2k-1}) - 21(4^{k+1})$	Either $4(4^{k+1} + 5^{2k-1})$ or $4f(k)$; $21(5^{2k-1})$ $25(4^{k+1} + 5^{2k-1})$ or $25f(k)$; $-21(4^{k+1})$	A1; A1
	$f(k+1) = 4f(k) + 21(5^{2k-1})$ or $f(k+1) = 25f(k) - 21(4^{k+1})$	**dependent on at least one of the previous accuracy marks being awarded.** Makes $f(k+1)$ the subject	dM1
	If the result is true for $n = k$, then it is true for $n = k+1$, As the result has been shown to be true for $n = 1$, then the result is is true for all $n \in \mathbb{Z}^+$.		A1 **cso**
	Note	Some candidates may set $f(k) = 21M$ and so may prove the following general results	

		• $\left\{ f(k+1) = 4f(k) + 21(5^{2k-1}) \right\} \Rightarrow f(k+1) = 84M + 21(5^{2k-1})$ • $\left\{ f(k+1) = 25f(k) - 21(4^{k+1}) \right\} \Rightarrow f(k+1) = 525M - 21(4^{k+1})$

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